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ORIGINAL PAPER

On some analogies of modern science with Plato's science in *Timaeus* and on Plato's influence on Kepler and Ptolemy

Francesco Di Giacomo[1]

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Abstract
The *Timaeus* of Plato is the only one of his Dialogues devoted to science. Among his readers we find Descartes, Boyle, Kepler, Heisenberg, Hermann Weyl and, more recently, Frank Wilczek. The aspects of the Democritean atomistic theory and those of Plato's geometrical atomism in *Timaeus* are discussed and compared. Plato presents the first mathematical theory of the structure of matter at three levels, analogous to the modern molecular, atomic and sub-atomic levels. In an impressive progress with respect to the Democritean theory with its unalterable micro-entities, Plato introduces in science the inter-transformability of elementary corpuscles and so the first "chemical" reactions in the history of science. Analogies with modern physics and chemistry are described throughout and Plato's influence on Kepler and Ptolemy is also treated. The different fortunes of Plato's geometric atomism and celestial motion in *Timaeus* are described in the Discussion section.

Keywords Early Greek science · Foundation of science · History of philosophy¸ History of science · Plato's *Timaeus* · Philosophy of science

1 Introduction

This article is the follow up of the one titled "Early theoretical chemistry: Plato's chemistry in *Timaeus*" recently published in *Foundations of Chemistry* and here cited in the references. In that article analogies of Plato's science with modern chemistry were dealt with.

Werner Heisenberg found "resemblance" between "the modern view" of elementary particles and "the elementary particles of Plato's *Timaeus*" because in both cases they are described by "mathematical forms" (Heisenberg, 1962, pp. 71, 72). Another resemblance between modern physics and Plato's physics for Heisenberg was the importance of symmetry in both: "In the beginning was symmetry! This sounded like Plato's *Timaeus*" (Heisenberg, 1972, p. 133). In this article and in the preceding one published in *Foundations of Chemistry,* I am extending Heisenberg's resemblances to other analogies between Plato's science and modern one. In the present article, analogies with modern physics are also considered. After a brief reminder in this introduction of the atomic theory of Leucippus and Democritus, Plato's geometrical atomism is expounded in the following couple of sections and then some analogies with modern physics are described. Finally, in the Discussion and Conclusions sections Plato's influence on Kepler and Ptolemy is also treated.

We all know about the atomic theory of Leucippus of Miletus (fl.435 BCE) and Democritus of Abdera (fl.410 BCE), a theory "rightly considered the culmination of Presocratic speculation" (Lloyd, 1970, p. 45). We just remind the readers that the word *atomon* in Greek means indivisible and that "The basic postulate of ancient atomism in its original, fifth century form, was that atoms and the void are alone real. The differences between physical objects, including both qualitative differences and what we think of as differences in substance, were all explained in terms of modifications in the shape, arrangement and position of the atoms… The atoms are infinite in number and dispersed through an infinite void. They are, moreover, in continuous motion, and their movements give rise to continuous collisions between them. The effects of such collisions are two fold. Either the atoms rebound from one another or, if the colliding atoms are hooked or barbed or their shape otherwise correspond to one another, they cohere and thus form compound bodies. Changes of all sorts are accordingly interpreted in terms

✉ Francesco Di Giacomo
francesco.digiacomo@uniroma1.it

1 Dipartimento di Scienze di base e applicate all'ingegneria, Sapienza Università di Roma, Rome, Italy

of combination and separation of atoms" (Lloyd, 1970, pp. 45–46). A more detailed discussion may be found in Lloyd's book. "*Ancient atomism* is important for two reasons. Firstly, it is the ancestor of modern atomism. Though the modern theory of the elements is far more sophisticated, ultimately its roots are with Leucippus and Democritus. Secondly, the ancient atomism was also the first properly thought out two-level theory of the world. It distinguishes between what humans perceive and how the world actually is at the atomic level. It introduces the idea that the reality behind appearances might be radically different from the appearances" (Gregory, 2001, p. 13). The ingenuity of the ancient atomists is astounding, we find here many words and concepts which modern physicists and chemists are familiar with, but what is clearly missing is a mathematical description. Without that, those beautiful, profound, genial intuitions don't allow quantitative descriptions and no deductions are possible.

In the fifth century the philosopher Empedocles of Acragas (492–432 BCE), today Agrigento in Sicily, "introduced the four *elements* of earth, water, air and fire which were to become standard in the Greek thought. These should not be understood as literally earth, water and so on, but in a more abstract manner, as solid, liquid, gas and fire, or as principles of solidity, fluidity, gaseousness and fieriness. Earth, water, air and fire had always been a Greek classification of matter, but it was Empedocles who took the bold step of insisting that these were the *four basic elements*. He also insisted that the physical objects were *fixed proportions* of these elements (Gregory, 2001, p. 28).

2 Plato associates the shapes of regular polyhedra to Empedocles' elements

After having set the stage in the above introduction, we are now ready for the description of Plato's *geometrical atomism*, a theory we find in *Timaeus*, the only dialogue Plato devoted to science, a dialogue which was for many centuries the most influential of Plato's works." It was not until the fourth century BCE, the century of Plato, that the Greek philosophy and mathematics reached their zenith" (van der Waerden, 1963, p. 106) and probably around 360 BCE Plato composed the dialogue *Timaeus* (Plato, p. xiv). Plato associated to the four elements of Empedocles the shapes of four of the five platonic solids: tetrahedron for fire, octahedron for air, icosahedron for water and cube for earth. "The tetrahedron was taken as belonging to fire because of its smallness (smallest number of sides), its sharpest corners, and its presumed high mobility. The cube was assigned to earth as being suitable for a stable surface. Air, being more mobile and lighter than water, was assigned the octahedron, while water was left with the icosahedron" (Benfey & Fikes, 1966). Why did Plato associate four convex regular polyhedra, the Platonic solids, to Empedocles' four basic elements? It was an aesthetic criterion, the beauty of those solids: "We have to decide, then, which are *the most beautiful bodies* that can be created. There should be four of them, and they must be dissimilar from one another but capable (in some cases) of arising out of one another's disintegration" (Plato 53e, p. 47).[1]

It may be of interest to our readers to have some supplementary information on the geometry of polyhedra and on the fundamental importance of symmetry in ancient and modern science. The geometry of regular polyhedra has been described by Hilbert and Cohn-Vossen (1999, pp. 89–93). Hermann Weyl writes in his book on symmetry that "Andreas Speiser has advocated the view that the construction of the five regular solids is the chief goal of the deductive system of geometry as erected by the Greeks and canonized in Euclid's Elements" (Weyl, 1952, p. 74). On symmetry he writes that "Beauty is bound up with symmetry" (Weyl, 1952, p. 3), he remarks though that "the Greeks never used the word "symmetric" in our modern sense. In common usage συμμέτρος means *proportionate*, while in Euclid it is equivalent to our *commensurable*: side and diagonal of a square are incommensurable quantities, ἀσύμμετρα μεγέθη'"" (Weyl, 1952, p. 75). Heisenberg writes, as we have seen, that "In the beginning was symmetry! This sounded like Plato's *Timaeus*" (Heisenberg, 1972, p. 133). Physics Nobel laureate Frank Wilczek recently wrote: "In its symmetry-based standard model, it would appear, fundamental physics comes closest to achieving the vision of Pythagoras and Plato: a perfect correspondence between what is real and what is mathematically ideal" (Wilczek, 2016).

3 Plato' elementary triangles

Plato denied to Empedocle's primary bodies the status of elements and explained their generation "from prior and *simpler* beginnings. He intended to construct the geometrical shapes of the four *primary bodies* from triangles which he takes as *elementary*" (Cornford, 1997, p. 162). After his decision to use the polyhedra to describe the four Empedocles' elements, Plato writes that "There are two basic triangles" (Plato, 2008, 56d, pp. 46–47). The two basic triangles are such that each of the six squares making up the cube is made up of two copies of one of the two basic triangles and each of the equilateral triangles making up the other three

[1] The numbers and letters that appear in the margins of the translations of *Timaeus*, like the 53e that we find here in the above reference, are the standard means of precise reference to passages in *Timaeus*, see (Plato, p. lviii.).

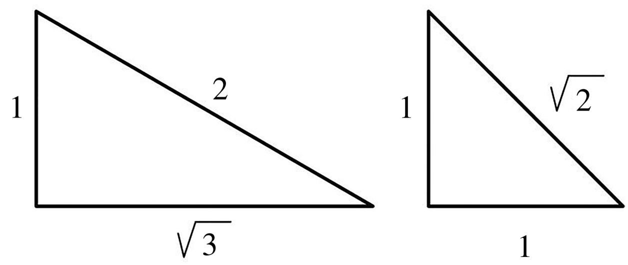

Fig. 1 Plato's elementary triangles

polyhedra is made up of two copies of the other basic triangle. The square face is made up of two copies of the basic triangle in which "the two acute angles are each half of a right angle" (Plato, 2008, 56d, pp. 46–47), the square face is hence made up by these right–angled isosceles triangles. The equilateral triangle is made up of two copies of the other basic triangle, a right–angled triangle in which "the square of the longer side is triple than the square of the shorter side" (Plato, 2008, 54b, p. 48) where Plato's "side" is here for cathetus, the square of the longer cathetus is the triple of the square of the shorter cathetus. Two of these right–angled scalene triangles make up the face of the equilateral triangle. The first basic triangle has sides of lengths, say, 1, 1, and $\sqrt{2}$, the second of lengths 1, $\sqrt{3}$ and 2, see Fig. 1. In Fig. 2 are shown the isosceles triangle and the square each made up by two elementary triangles. The basic triangles are Plato's elementary or *stoicheic* triangles. The word *stoicheion*, (στοιχεῖον in Greek) means "letter" as element of the syllable, or "element", "principle" in the physical things, science, etc.. Euclid was called στοιχειωτής, "teacher of elements", (translated in English from Rocci (1979, p. 1709)).

In Plato (2008, 56d,e, p. 51) we find four Plato's "chemical reactions" written in words. Giving the equilateral triangle the symbol T and writing them using the modern chemical symbolism, the reactions are:

$$T_{20} = 2T_8 + T_4 = 5T_4 \quad \begin{array}{l} \text{1 icosahedron} = \text{2 octahedra} + \text{1 tetrahedron} = \text{5 tetrahedra} \\ \text{1 water} = \text{2 air} + \text{1 fire} = \text{5 fire} \end{array} \tag{1}$$

$$2T_4 = T_8 \quad \begin{array}{l} \text{2 tetrahedra} = \text{1 octahedron} \\ \text{2 fire} = \text{1 air} \end{array} \tag{2a}$$

$$T_8 = 2T_4 \quad \begin{array}{l} \text{1 octahedron} = \text{2 tetrahedra} \\ \text{1 air} = \text{2 fire} \end{array} \tag{2b}$$

$$2.5\ T_8 = T_{20} \quad \begin{array}{l} \text{2.5 octahedra} = \text{1 icosahedron} \\ \text{2.5 air} = \text{1 water} \end{array} \tag{3}$$

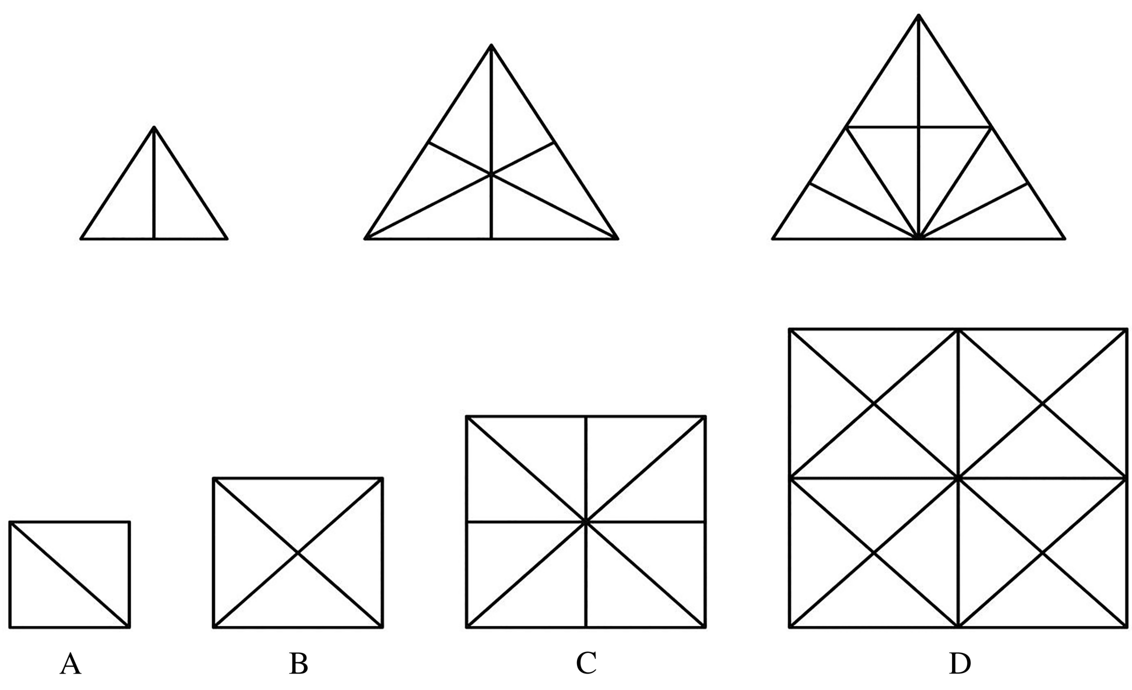

Fig. 2 Graded sizes of equilateral triangles and squares

We see that the Platonic solids representing fire, air and water are *analogs of molecules,* in this case of a cage or cluster type (Cotton & Wilkinson, 1988, pp. 18ff). The equilateral triangles are analogs of atoms and, as we have seen above, the atoms can be decomposed in elementary right-angled triangles which are analogs of sub-atomic structures. Plato's equations are the first "chemical" equations in history. The analogy with some chemical equations is so close as to be an isomorphism, compare with reactions such as $2I_2 = I_4$ (Cotton & Wilkinson, 1988, p. 584), $I^- + I_2 = I_3^-$ (Cotton & Wilkinson, 1988, p. 453), $3O_2 = 2O_3$ (Partington, 1961, p. 657).

We see that going from reactants to products the atoms are *reshuffled but conserved*, an idea that appears here 2200 years before John Dalton and Amedeo Avogadro. A detailed discussion of the above "reactions" can be found in Di Giacomo (2020). The reader could ask if we are justified in calling the above reactions "chemical equations". To answer this question, I quote from (Brock, 1993, pp. 118, 119) where we read that Lavoisier in one of his essays wrote that "In order to show at a glance the results of what happens in the solution of metals, I have constituted formulae of a kind that could at first be taken for algebraic formulae, but which do not have the same object and which do not derive from the same principles.". Brock comments that "The important point here was that Lavoisier used symbols to denote both constitution and quantity. Although he did not use an equal sign, he had effectively hit upon the idea of a chemical equation. As we shall see, once the Berzelius' symbols became firmly established in the 1830s, chemists began immediately to use equations to represent chemical reactions." On pp.156 and 157 of the same book, Table 4.2 nicely describes the development of the concept of chemical equation. After this brief history of the term "chemical equations" I believe I am justified to use the term chemical equations because in Plato's reactions we see that the atoms are reshuffled but conserved, we see in Plato's reactions their conservation in nature and numbers, in other words Lavoisier's constitution and quantity.

The stability of atoms in the chemical reactions motivated Niels Bohr in his study of the atomic structure: "My starting point was not at all the idea that an atom is a small-scale planetary system and as such governed by the laws of astronomy. I never took things as literally as that. My starting point was rather the stability of matter, a pure miracle when considered from the standpoint of classical physics" (Heisenberg, 1972, p. 39). Plato of course did not know the concept of mass and of mass being conserved in a reaction, as established by Lavoisier in 1789. What is here conserved is the *total area* of the atoms, we find here an interesting fascinating isomorphism between *mass* and *area*.

We introduce here a last concept of Plato's geometrical atomism. Plato considers "atoms", equilateral triangles and squares, of *different sizes*, see Fig. 2 where different *grades of size* are shown built from the two elementary triangles. In chemistry, we have atoms of the same nature but of different masses, the isotopes. Plato considers atoms of the same geometrical form, i.e. of the same nature, but of different *areas*, we find here again the isomorphism between *mass* and *area.* Friedländer (1958) has rightly called "*isotopes*" these equilateral triangles and squares of different sizes. Plato admits the transformability between different grades, in Di Giacomo (2020) the example is reported of reaction (4) there in which 4 molecules of earth of grade A can be transformed in 1 molecule of earth of grade C. From all of the above, we can find other analogies with modern physics and chemistry. In Plato's geometrical atomism the *four* primary bodies, the *four* platonic solids, are, as we have seen, the analogues of molecules while their faces are "planar atoms" (Wilczek, 2015, p. 39). These *two* "atoms", the equilateral triangle and the square, are really *compound* atoms like modern atoms are. We have also seen that Plato considers, at a lower structural level of matter, *two* right-angled triangles in which the equilateral triangles and the squares can be divided. These *stoicheic,* i.e. *elementary* triangles are Plato's analogues of sub-atomic particles. Note that Plato's sub-atomic particles are the simplest *right-angled* triangles which are mathematically elementary in the sense that they are built up using the two smallest entire numbers and the two smallest irrational numbers from entire numbers. The choice of those triangles bears witness of Plato's genius. We have seen that we have *four* different geometrical forms that are the analogues of molecules, the Platonic solids, but, going down in structures, only *two* geometrical forms, the equilateral triangle and the square are the analogues of atoms, finally ending up at the lowest structural level with only *one* kind of polygon, the right-angled triangle, even if Plato considers two kinds of them, see Fig. 1. In chemistry one goes from an enormous number of molecules to only 92 atoms (not considering the transuranic elements) and down to only three elementary particles making up all of them, electrons, protons and neutrons but with different nuclear and electronic structures for the different atoms. We find here still another analogy because the right-angled triangle comes in two different forms in Plato's atoms.

We find two puzzling and surprising analogies of Plato's geometrical atomism with modern particles physics. Plato's elementary triangles, his analogs of modern elementary particles, are extended over a finite area, they are not point-like particles, a feature reminding of other non-point-like particles like the one-dimensional strings of String Theory. We have, moreover, two elementary triangles and analogously we have two quarks, up and down ad two leptons, electron and electronic neutrino in the first generation of elementary particles. Other couples of quarks and leptons are in the other two generations of elementary particles. But the analogy can be carried further: the up and down quarks have charges equal to + 2/3 and − 1/3 of the electronic *charge*, the side *lengths* of the stoicheic triangles are given by the numbers 1,2, $\sqrt{2}$ and $\sqrt{3}$, the first three entire numbers are used in both cases, in one case for charges, in the other for lengths. Of course the above analogies are just interesting analogies and no more than that.

4 Physical interactions and properties

"The gist of the mathematical, combinatorial part of the theory…is supplemented by another section on *the mechanics of the transformation*…Plato denies action at a distance and reduces all physical interactions to operations involving *contact*-ultimately pushing, cutting and crushing. Fire corpuscles cut air and water corpuscles, for the solid angles of tetrahedra are smaller—hence sharper-than those of octahedra or icosahedra; for similar reasons air corpuscles cut water corpuscles. Here we see air changing into fire, and water changing into fire or into air or both. The converse transformations will occur when a small quantity of fire is enveloped in a larger mass of air or of water, or a small quantity of air in a larger mass of water; then the bigger mass squeezes and smashes the polyhedra of the smaller one; we then see fire turning into air or into water or both, or air turning into water (Vlastos, 2005, pp. 233, 234). See the above reactions in Di Giacomo (2020).

We have seen that, contrary to Leucippus, Democritus and Descartes (Gregory, 2000, pp. 233–234), Plato considers a limited number of atoms, another characteristic in common with modern science. "The critical differences between Plato and the atomists …are that where the atomists postulate unlimited shapes and sizes of ultimate particles, Plato postulates *a small number* restricted by teleological principles, postulates *structured* particles, and sees the need for the teleological ordering of both the extrinsic and intrinsic organization of the ultimate particles" (Gregory, 2000, p. 233). "For Descartes the ultimate particles of matter are three-dimensional, come in an indefinite number of shapes and sizes and are indefinitely divisible, all contrary to Plato's account… Plato and Descartes appear to be in strong agreement on the inherent properties of matter. *Matter has geometrical properties and motion, and no other properties.* So for Descartes interaction between particles is by *contact action* only, and this serves to explain all attractive and repulsive effects that might be thought to operate at a distance, including the hard cases of gravity, electricity and magnetism. Plato is keen to stress this as well, at *Timaeus* 80 b8–c4: 'And moreover the flowing of waters, the fall of thunderbolts and the wonderful attraction effects of electricity and the magnet, all these are not due to any power of attraction, but to the fact that there is no void and that the particles push into each other'" (Gregory, 2000, p. 234).

We may ask: how about the interior of Plato's "molecules", the polyhedra, the primary bodies? The particles of the primary bodies are not simply portions of empty space. "Plato differs from Democritus in that his particles are not impenetrable solid lumps, but contain 'motions (changes) and powers'" (Cornford, 1997, pp. 261–262). "What is the 'matter' of which these plates are composed? And is the interior hollow in the sense of an absolute vacancy? Plato does not say so, but speaks of the contents of the figures as qualities or 'motions and powers'. The whole description of the warfare of the primary bodies in the process of transformation implies that these powers are actively operating. Without them, the geometrical figures could not move at all or break one another down" (Cornford, 1997, p. 229). Plato's molecules with their "active power (*δύναμος τοῦ ποιεῖν*)" (Cornford, 1997, p. 226), allows them to react as we have seen above. A further characteristic of Plato's geometrical atomism in common with modern science and different from that of the ancient atomists is that his microscopic molecules, even if not perceptible to the senses, have nonetheless characters in common with macroscopic objects. And so just as electrons have charge and mass, properties that we find also in macroscopic objects, Plato's microscopic tetrahedra have a cutting property in common with a knife and "the interaction of tetrahedra with our bodies gives rise to the qualities we associate with fire" (Gregory, 2000, p. 224).

"If we ask why the four bodies are arranged in this order rather than in any other order, the answer may perhaps be found in statement (Plato 59B) that, of the three bodies formed of the same elementary triangle, fire is the most *mobile,* 'the *lightest,* as being composed of the smallest number of similar parts', air stands next, and then water" (Cornford, 1997, p. 265). We have here a premonition of the kinetic theory of gases with its velocities of gases depending on mass, the velocity at a certain temperature being higher for gases of smaller mass (Feynman, Leighton, Sands 1963, Volume 1, p. 9). Here the "mass" depends on the area, on the number of constituent triangles.

"What about movement and rest?… it's difficult, not to say impossible, for there to exist something to be moved if there's no mover for it, or for a mover to exist if there's nothing to be moved. In the absence of a mover and a moved, there's no such thing as motion, and mover and moved cannot possibly be uniform with each other. It follows that we should always associate *rest* with *uniformity* and attribute *motion* to *diversity*. And diversity is due to inequality" (Plato's Timaeus 2008, 57e, 58e, pp. 52–53). We find here an analogy of the "mover" with the force and of the rest as due to zero potential difference.

"Admittedly, Plato did not permit a vacuum while Democritus did. Quantum theory, however, can accommodate either view regarding vacua" (Benfey & Fikes, 1966). Newton in his tenets about atoms was a Democritean, from what we can read in Newton, Opticks (1952, p. 400): "It seems to me that God in the beginning formed matter into solid, massy, hard impenetrable movable particles…". Plato was then much closer than Newton to modern physics and chemistry, nonetheless they both used geometry in their theories, and on geometry Newton wrote: "It is the glory of geometry that from a few principles, brought from without, it is able to produce so many things" (Iliffe & Smith, 2016, p. 345).

5 A seemingly arbitrary feature

We have encountered a seemingly arbitrary feature which has "never been satisfactorily explained" (Cornford, 1997, p. 217): Why does Plato divide the equilateral and the square into triangles and why these triangles are "the fairest"? Cornford's explanation: the elementary triangles are the fairest "because of a certain property possessed by both elementary triangles, which we can now see to be one of the reasons why they are the 'best' that could be selected. It is a property that could be regarded as characteristic of an 'element': either of the two triangles can be subdivided without limit into parts of the same type as itself" (Cornford, 1997, p. 233). I surmise, that "Plato wishes to provoke his readers to think for themselves" (Gregory, 2000, p. 255), and that the reason why he called his triangles "the most beautiful" (Plato, 2008, 54a, p. 47) might also have been that:

1. Plato uses as atoms the two *simplest regular polygons.* The fairest triangles are such because they are the halves of his two geometrical atoms, the two triangles composing equilateral and square being the most simple *common structure* between his atoms one might imagine.
2. The ratios of the lengths of the triangles' sides are expressed in terms of the two first integers, 1 and 2, and the two first irrational numbers obtained from square roots of integers, $\sqrt{2}$ and $\sqrt{3}$, which are the lengths of the first two hypotenuses of the right– angled triangles in Theodorus' spiral, a characteristic that has probably been appealing to Plato's sense of beauty.
3. Because "The [musical] intervals of an octave, fifth and fourth could all be expressed in terms of simple numerical ratios, 1:2, 2:3, and 3:4." (Lloyd, 1970, p. 26), and the fact that the numbers used by Plato are all square roots of 1, 2, 3 and 4, may have also been appealing to him.
4. There might be a further possible reason. At *Timaeus* (33b) we read: "…similarity being, in his opinion, incomparably superior to dissimilarity" (Plato's Timaeus 2008, p. 21), we should also remember "the Greek obsession with symmetry" (Vlastos, 2005, p. 38), that "the symmetry properties always constitute the most essential features of a theory" (Heisenberg, 1962, p. 133) and that Mendeleev and predecessors (Partington, 1961, pp. 172 ff.) tried to find common characters among different atoms. It is then tempting to guess that Plato in search for similarity ("sameness and difference, likeness and unlikeness", as he would say in *Theaetetus* (Gregory, 2000, p. 246), did the same, even if of course his similarity was one of possible common geometrical characteristics and not a similarity of chemical properties. It has been a common purpose of some of the most eminent scientists that of looking for unifying theories or principles. Plato had clearly a similar attitude. May we imagine that he was looking for something common between the geometrical structures of his two atoms, the equilateral and the square? One obvious common character of the equilateral triangle and of the square is that they have centers of symmetry. Evidently this is rather trivial, it is "hardly relevant" in itself (Cornford, 1997, p. 217). But if you want to draw attention to the centers while at the same time using the two basic triangles to describe the equilateral triangle and the square, then the simplest way to do it is that shown by the equilateral and the square of grade size B in Fig. 2. Here we find another common structural feature: all of the elementary triangles in the equilateral triangle or in the square are related to the common center, all of them having one common vertex in that center. Here we find another analogy: modern atoms have a center, the nucleus, common for all the electrons which are related to the center. By the way, Plato's subdivision of grade B equilateral in Fig.2 is a particular case of the *barycentric subdivision* of a 2–dimensional complex K^2 in topology (Alexandroff, 1958, pp. 45–46).

The atoms of modern physics have common features: all of them have nuclei and electrons. Plato considers two subatomic particles, the two elementary *stoicheia* of minimal size, his *irreducible elements*, which, being both of a triangular nature, planar polygons with the minimum number of sides, are more similar to each other than the two compound atoms, one of which is a triangle and the other a square. A character of triangles that suggests their rightly being the irreducible elements in geometrical atomism is also that they are inherently *rigid*, in the sense of having *structural rigidity*, i.e. not being deformable without changing the lengths of the sides, they cannot be altered by any force applied to them. A square is not rigid because it can be tilted over into a parallelogram but Plato's square atoms are rigid because they are made up by two isosceles squared angle triangles which are rigid. Did Plato have an intuition of this structural rigidity? It is interesting anyway that the *unalterable solidity* of the Democritean atoms has evolved into the geometrical *structural rigidity* of Plato's atoms. The hardness of modern atom first appeared with that of its nucleus in Rutherford's scattering experiments and then with the *quantum* structural "rigidity" of Bohr's atom. The electronic states are "rigid" compared with the classical planetary orbits in the sense that, as Weisskopf writes: "Electrons can assemble around the nucleus only in a few well-defined modes—the quantum states—and not in others" (Weisskopf, 1972, p. 57), there are no intermediate levels in Bohr's quantum model, as shown in the Franck–Hertz experiment. And the quantum states are "rigid" in the sense that they show "stability against collisions" (Weisskopf, 1972, p. 57): the mercury atoms in that experiment behave as "hard" when slower electrons merely bounce them off without losing any kinetic energy. The elastic scattering of slow enough electrons off mercury atoms shows that atoms *can* be hard, as Newton surmised. Plato's theory appears then not only the oldest molecular theory but also the oldest atomic theory.

6 Plato's science in Timaeus versus modern science

Felix Klein in his assessment of the "Elements" of Euclid distinguishes the *historical importance* of the work of the ancients from that of *permanent importance* (Klein, 2004, p. 195). In the case of Plato among the topics of historical importance are the "theology and speculative metaphysics" (Vlastos, 2005, p. 51), the "many fanciful and arbitrary elements in Plato's doctrine" (Vlastos, 2005, p. 77), his "metaphysical fairy tale" (Vlastos, 2005, p. 65). One should consider though that "The scientific theory of celestial motions suggested to Plato by his metaphysical scheme conveyed genuine insight (Vlastos, 2005, p. 63). Which ones are then the characteristics of Plato's *Timaeus* that are of *permanent importance* and that we today find in modern science? Here is a brief list of them:

1. The great importance of *beauty* in scientific theories: As for Plato's theories in *Timaeus* "It would be hard to think of a physical theory in which aesthetic considerations have been more prominent" (Vlastos, 2005, p. 93), but Gregory Vlastos writes that the beauty principle had already been implicitly introduced in science by the *Physiologoi* when they created the concept of *kosmos*: "In English, *cosmos* is a linguistic orphan, a noun without a parent verb. Not so in Greek which has the active, transitive verb, *kosmeō*: to set in order, to marshal, to arrange. It is what the military commander does when he arrays men and horses for battle; what a civic official does in preserving the lawful order of a state; what a cook does in putting foodstuffs together to make an appetizing meal …What we get in all these cases is not just any sort of arranging, but one that strikes the eye or the mind as pleasingly fitting: as setting, or keeping, or putting back, things in their proper order. There is a marked aesthetic component here, which leads to a derivative use of *kosmos* to mean not order as such, but *ornament*, *adornment*; that survives in the English derivative, *cosmetic*, which, I dare to say, no one, without knowledge of Greek, would recognize as a blood-relation of *cosmic*. In the Greek the affinity with the primary sense is perspicuous since what *kosmos* denotes is a crafted, composed, beauty-enhancing order" (Vlastos, 2005, p. 3).
2. The first consistent physical and chemical systematic theory built on *the most advanced mathematics of the time*, that of the irrational numbers and of the regular polyhedra. The theory, as we have seen, describes the matter at three microscopic, non-perceptible levels, the equivalents of modern sub–atomic, atomic and molecular levels, "The demiurge creates spatially limited entities, the stoicheic triangles, which move around in space and combine with each other to form more complex entities, such as the four elements" (Gregory, 2000, p. 224).
3. Plato, as we have seen, uses the first two irrational numbers from entire numbers, $\sqrt{2}$ and $\sqrt{3}$, the irrationality of the first was already known to the Pythagoreans, that of the second had been demonstrated by his friend and master Theodorus of Cyrene (van der Waerden, 1963, p. 110), the discoverer of the Theodorus' spiral (van der Waerden, 1963, p. 142). Two of the regular poyhedra, the octahedron and the icosahedron, had been discovered by Plato's pupil Theaetetus (van der Waerden, 1963, p. 173). "Plato himself emphasized the necessity of mathematics as training for the mind in pursuit of forms; over the door of his Academy he is said to have inscribed: 'Let no one destitute of geometry enter my doors'" (Kuhn, 1957, p. 128). He studied mathematics with the great mathematicians Architas of Tarentum and Theodorus of Cyrene; Eudoxus of Cnidus, Theaetetus and Heraclides of Pontus were his pupils (van der Waerden, 1963). "Popper comments that one of Plato's main contributions is that 'Ever since but not before, geometry (rather than arithmetic), appears as the fundamental instrument of all physical explanations and descriptions, in the theory of matter as well as in cosmology'" (Gregory, 2000, p. 240).
4. The introduction in science of the idea of *plausibility*: "We'll have to be content if we come up with statements that are plausible…we ought to accept the likely account… (Plato, 2008, 29c, p. 18).

5. The importance of theoretical sciences in *education*: "The *Timaeus* is very clear on the benefits of the study of astronomy. By studying the visible revolutions of god, i.e. the heavens, we will help to stabilize the *revolutions in our own heads* and so become *rational* and knowledgeable. That will of course help with the standard Platonic project of becoming like god. So too a standard motif in Plato is that studying theoretical sciences is immensely important for the education of the young" (Gregory, 2000, p. 269) because "if proper nurture is supported by education, a person will become perfectly whole and healthy" (Gregory, 2000, p. 34).
6. *Timaeus* is the first book in history in which we find characteristics common with modern science. In particular some analogies with modern physics and chemistry have been discussed. Of course such analogies with modern science, suggestions, resonances and hints of things to come are intended to be neither historical interpretations of Plato's own views, thoughts and programs nor explanations of *Timaeus* with its many obscurities, ambiguities and inconsistences and with its relations to other Plato's dialogues and to works of other ancient Greek philosophers, fields better left to the learned commentaries of Platonists. I am encouraged in this approach of *searching for analogies* by the fact that "Plato's usual aim in writing is not so much to impart knowledge, but rather to attempt to generate understanding, and so in the main his purpose is dialectical rather than dogmatic and that he attempts to create some sort of interaction between the reader and the text" (Gregory, 2000, p. 255). Plato "is forcing the readers to do some thinking for themselves" (Gregory, 2000, p. 256) and "If I am right about Plato's strategy… then he is asking his readers to do quite a lot in terms of thinking for themselves and interacting with the various problems that the *Timaeus* might set" (Gregory, 2000, p. 260).
7. Finally, *Timaeus* is also aware of the importance of *consistency* and *coherency* of a theory because "he warned us that his account may not be entirely consistent" (Plato's *Timaeus* 2008, p.li). Moreover in *Timaeus* we find that pursuit of and desire for *systematization* that is for Chandrasekhar the fundamental driving force of scientists (Chandrasekhar, 1987, p. 13).

7 Discussion

It is interesting to compare the very different fortunes of Plato's astronomy and that of Plato's theory of matter as are both described in *Timaeus*. Plato's geometrical atomism is largely ignored in the education of physicists and chemists. Platonist Andrew Gregory believes that this is a shame "as geometrical atomism is an important and interesting theory in itself, as well as having important relationships to the earlier theories of Leucippus and Democritus and later theories such as those of Descartes" (Gregory, 2000, p. 187). "Geometrical atomism postulates a small number of *well–defined* ultimate particles, allows *microstructures*, gives *prominence to mathematics* and has a *structured system of bonding*" (Gregory, 2000, p. 273) and "If we look for affinities to modern science with Plato, then most strongly of all there is the use of mathematics wherever possible" (Gregory, 2000, p. 273). This complaint reminds of J.S. Bell's complaint that the De Broglie–Bohm theory of hidden variables is not taught to students of quantum mechanics (Bell, 1987, p. 171).

"Plato's major contributions to astronomy are the first model of the heavens with an offset ecliptic/zodiac, and the principle that the motions of all heavenly bodies can be resolved into combinations of regular circular motions" (Gregory, 2000, p. 270). "Plato recognized that there are no irregular motions in the heavens and, in supposing that all motions of the heavenly bodies are either simple regular circular motions or *combinations* of regular circular motions, he set the parameters for one of the longest and most fruitful research programs in the history of science. The concentric–spheres astronomies of Eudoxus, Callippus and Aristotle all developed from this, as did the epicyclic astronomies of Ptolemy and his followers. Even as late as 1543, Copernicus, supposing the earth to be in motion around the sun, stayed with combinations of regular circular motions. It was not until 1609 that Kepler suggested that planetary orbits are simple ellipses about the sun" (Plato, 2008, p. xl). Plato's astronomy enjoyed an enormous success, first by the genial work of his own pupil Eudoxus and Eudoxus' pupil Callippus. His was a "purely kinematical model which aimed to show how, if certain motions were assumed, the mathematically deduced consequences would save the phenomena. This was to be the way of the future—the road traveled by Eudoxus, Apollonius, Hipparchus, Ptolemy" (Vlastos, 2005, p. 65). "This is how Ptolemy celebrates that journey's end: 'Now that we are about to demonstrate in the case of the five planets, as in the case of the sun and the moon, that all of the phenomenal irregularities result from *regular and circular* motions—for such befit the nature of divine beings, while disorder and anomaly are alien to their nature—it is proper that we should regard this achievement as a great feat and as the fulfillment of the *philosophically grounded* theory [of the heavens]'" (Vlastos, 2005, p. 65).

"One of the general weaknesses of ancient science is that many of its practitioners presented their theories as finished and dogmatic systems….it is a modern rather than an ancient response to say 'We don't know yet, there are problems but we hope to find an answer along the following lines'. One very important exception to this may be Plato and the tradition he begins in astronomy. The weaknesses of the astronomical model of the *Timaeus*, which are considerable and greater than is generally recognized, are offset by the methodological and cosmological insights, and the fact that the model is (and is intended to be) the *prototype* for a whole new tradition of astronomy and cosmology" (Gregory, 2000, p. 158). In other words "A criticism that might be levelled at much of Greek science is that many of the contributors presented their work as the final word on the matter and in a finished system. It is very rare to find anyone leaving possibilities open or indicating that further research along certain lines would be fruitful. In this sense, Greek science in general tended to be dogmatic. Plato, to some degree at least, can be absolved of this, especially if we consider the *Timaeus* to be more about how to do cosmology than specific cosmology itself. One can perhaps see this in the presentation of geometrical atomism at Plato (2008, 54a, p. 47), where anyone who could give an argument for better basic entities than the two types of triangles would be welcomed as a friend rather than a foe. Plato was well aware of many of the deficiencies of the very simple astronomical model he puts forward in the *Timaeus*, and so could not have meant this model to be taken literally or dogmatically" (Gregory, 2000, p. 266). "The astronomy of the *Timaeus* is best viewed as a prototype rather than a completed and definitive view. So perhaps the *Timaeus* (at least in part) sets out to pose its readers some problems" (Gregory, 2000, p. 241).

"Kepler was an ardent Platonist, and an avid reader of *Timaeus*.... Kepler attempted to derive the size and number of the planetary orbits from the platonic solids" (Plato, 2008, pp. xxix, xxx). But, as Owen Gingerich writes on a recent book review, "it was Kepler's brilliance to try to find a geometrical explanation, nutty as the explanation might appear to us today. Tycho Brahe had also thought about using regular polyhedra in some way, and hence the idea resonated with him when Kepler's letter looking for a job arrived. Without his crazy idea, maybe Kepler would never have got access to Tycho's fabulous observations of Mars!" (Gingerich, 2016). I want to remark that Kepler's use of Platonic solids was not overlooked after his introduction of the elliptic orbits. As Thomas Kuhn writes in (1957, pp. 218, 219), "Kepler's use of the regular solids was not simply a youthful extravagance, or if it was, he never grew up. A modified form of the same law appeared twenty years later in his *Harmonices of the Worlds*, the same book that propounded the Third Law…Certainly the scientific attitude demonstrated in those of Kepler's "laws" which we have now discarded is not distinguishable from the attitude which drove him to the three Laws which we now retain." If we look for the topic "*Mysterium Cosmographicum*" in English *Wikipedia*, on p.2 we read that "Though the details would be modified in light of his later work, Kepler never relinquished the Platonist polyhedral–spherical cosmology of *Mysterium Cosmographicum.* His subsequent main astronomical works were in some sense only further developments of it, concerned with finding more precise inner and outer dimensions for the spheres by calculating the eccentricities of the planetary orbits within it. In 1621 Kepler published an expanded second edition of *Mysterium*, half as long as the first, detailing in footnotes the corrections and improvements he had achieved in the 25 years since its first publication".[2]

Plato's influence was not limited to that on the two great astronomers above, he also influenced Eudoxus "known as the father of scientific astronomy", as Carl B. Boyer's writes in *A History of Mathematics* (Boyer, 1980, p. 134). Through Eudoxus Plato's influence passed to Menaecmus who "discovered the curves that would be later known as ellipse, parabola and hyperbola." (Boyer, 1980, p. 135). Boyer writes that "Plato is important in the history of mathematics largely because of his role of inspirer and guide of other mathematicians." (Boyer, 1980, p. 124) and "his enthusiasm for mathematics made him famous not as mathematician but as creator of mathematicians" (Boyer, 1980, p. 122).[3] It is also possible an influence of Plato on the alchemists because, as Cornford writes and is reported in Di Giacomo (2020), "It would be interesting to know whether the alchemists were encouraged by his theory to attempt the transformation of metals" (Cornford, 1997, p. 252).

8 Conclusion

Dante in the Inferno (Hell), canto iv of his *Divina Commedia*, called Aristotle "*Il maestro di color che sanno*" (the master of those who know). Plato, following in the steps of his generous unenvious Demiurge, invited other researchers to go beyond his own theories in astronomy and in the structure of matter. In astronomy, "Simplicius attributes to a writer named Sosigenes (second century CE) …that Plato put this question to students of astronomy: 'By the assumption of what uniform and orderly motions can the apparent motion of the planets be accounted for?'" (Lloyd, 1970, p. 84) also reported in Kuhn (1957, p.55) and Vlastos (2005, pp. 59, 60). And we have seen above the invitation to find triangles fairer than his own. It is maybe because of this open generous attitude towards collaboration with other researchers that the Platonist Auguste Diès has rightly called Plato "*le maître de ceux qui cherchent*" (the master of those who undertake the quest) (Diès, 1922, Tome II, p. 299). I finally report the opinion of Alfred North Whitehead on Plato's influence on European philosophical tradition: as we have seen in *Timaeus*, Plato's writing is "an inexhaustible mine of suggestion" (Whitehead, 1978, p. 39).

[2] See, J.V. Field's book *Kepler's Geometrical Cosmology*, Chapter IV, pp.73ff.

[3] The preceding quotes are translated in English from the Italian edition Boyer 2020.

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